NONDESTRUCTIVE THERMOPHYSICAL-PARAMETER MONITORING FOR VARIOUS EQUIPMENT STATES

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Nondestructive monitoring methods are proposed for the thermophysical characteristics of planar specimens subject to various initial conditions (temperature, pressure, and water and mass contents), together with a method of selecting the preferred technique and the best instrument design.

Routine measurements on thermophysical characteristics make it necessary to devise special methods and automated instruments giving elevated reliability, which include nondestructuve test methods [1-5] and automated research systems [6].

Figure 1 shows a model for these methods in a system composed of three planar bodies, one of which is the test one. The following conditions are assumed: the test body is in the part $L_s < x < L_t$, while the first and second standard bodies correspondingly are in the parts $0 < x < L_s$ and $0 > x > -L_1$. The heater has output Q(t), which is supplied to the plane x = 0 of contact between the two standard bodies. The heat fluxes $q_s(t)$ and $q_1(t)$ going to the upper and lower standard bodies are related by

$$Q(t) = q_1(t) + q_s(t). \tag{1}$$

At the boundaries $x = L_t$ and $x = L_1$, there is a constant temperature U_b = constant, which is equal to the initial temperature for the entire system $U_b = U(x, 0)$ = const; known values are available for the thermophysical parameters of the standard specimens: thermal diffusivity a_s and a_1 and thermal conductivity λ_s and λ_1.

We assume that those quantities are constant over narrow temperature and pressure ranges; an important requirement is that $q_s(t)$ and $q_1(t)$ should be minimal, as they should cause temperature changes such that one can neglect the temperature dependence of the thermophysical coefficients.

The subscripts to the parameters are as follows: t for the test body, s for the upper standard specimen, and 1 for the lower one.

With these conditions, the temperature patterns $U \equiv U(x, t)$ are defined by the following:

$$\frac{\partial U_t}{\partial t} = a_t \frac{\partial^2 U_t}{\partial x^2}, \quad L_s < x < L_t \quad t > 0; \tag{2}$$

$$\frac{\partial U_s}{\partial t} = a_s \frac{\partial^2 U_s}{\partial x^2}, \quad 0 < x < L_s, \quad t > 0; \tag{3}$$

$$\frac{\partial U_1}{\partial t} = a_1 \frac{\partial^2 U_1}{\partial x^2}, \quad -L_1 < x < 0, \quad t > 0; \tag{4}$$

$$U_t(x, 0) = U_s(x, 0) = U_1(x, 0) = 0; \tag{5}$$

$$\lambda_t \frac{\partial U_t}{\partial x}\bigg|_{x=L_s+0} = \lambda_s \frac{\partial U_s}{\partial x}\bigg|_{x=L_s-0}; \tag{6}$$

$$\lambda_s \frac{\partial U_s}{\partial x}\bigg|_{x=0+0} = -q_s(t); \tag{7}$$

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TABLE 1. Errors in Nondestructive Methods

Methods of determining heat-flux characteristics	Error in determining $\Phi(g_t)$, %	Errors in determining characteristics, %	
		thermal diffusivity a	thermal conductivity λ
№ 1, known $Q(t)$, $U_s(l_s, t)$	14,77	14,80	10,84
№ 2, known $Q(t)$, $U_1(l_1, t)$	11,65	11,67	8,72
№ 3, known $Q(t)$, $U_s(l_s, t)$, $U_1(l_1, t)$	9,76	9,77	7,14
№ 4, known $U_1(l_1, t)$, $U_s(l_s, t)$	12,87	12,86	9,32

$$\lambda_s \frac{\partial U_1}{\partial x}\Big|_{x=0-0} = q_1(t); \tag{8}$$

$$U_t(L_t, t) = U_1(-L_1, t) = 0. \tag{9}$$

To simplify the theory and experiment, by U(x, t) we mean the temperature excess over the initial value, so (5) and (9) can be set as zero.

The method is based on integral characteristics [7], which give very simple working formulas for the thermophysical ones.

We use time integral characteristics for the temperature

$$U^*(p, x) = \int_0^\infty \exp(-pt)\, U(x, t)\, dt \tag{10}$$

and the heat flux

$$q^*(p) = \int_0^\infty \exp(-pt)\, q(t)\, dt, \quad p > 0. \tag{11}$$

The variable p in (10) and (11) is taken as a real positive number; we put $B = B(p, a) = \sqrt{p/a}$, and the solution to (2)-(9) for each of the contacting bodies gives the time integral temperature characteristics:

a) for the test body $L_s < x < L_t$:

$$U_t^*(x, p) = q_{sL}^*(p) \frac{\mathrm{sh}[B_t(L_t - x)]}{\lambda_t B_t \mathrm{ch}[B_t(L_t - L_s)]}; \tag{12}$$

b) for the upper standard body $0 < x < L_s$:

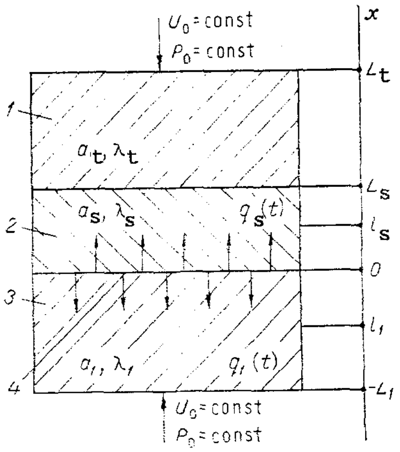

Fig. 1. Model for a contacting-body system in nondestructive thermophysical characteristic determination: 1) test body, parameters a_t, λ_t, and U_t; 2) upper standard body having a_s, λ_s, and U_s; 3) lower standard body, with a_1, λ_1, and U_1; 4) heater; P_0 pressure maintained constant during experiment.

TABLE 2. Set of Measuring-Instrument Functional States

Methods of determining heat flux characteristics	w_1	w_2	w_3	w_4
Viable states set for measuring instrument	$H_{w_1} = \{h_0\}$	$H_{w_2} = \{h_0\}$	$H_{w_3} = \{h_0, h_{ts}, h_{t1}, h_Q\}$	$H_{w_4} = \{h_0\}$

Note: h_0 normal operating state, h_{ts}, h_{t1}, h_Q correspondingly states of PS failure in temperature measurement at $x = \ell_s$, $x = \ell_1$, and in the heater heat-flux density meter.

$$U_s^*(x,\ p) = \frac{q_s^*(p)\,\mathrm{ch}\,[B_s(L_s - x)] - q_{tL}^*(p)\,\mathrm{ch}\,(B_s x)}{\lambda_s B_s \mathrm{sh}\,(B_s L_s)}; \tag{13}$$

c) for the lower standard body $-L_1 < x < 0$:

$$U_1^*(x,\ p) = \frac{q_1^*(p)\,\mathrm{sh}\,[B_1(L_1 - x)]}{\lambda_1 B_1\,\mathrm{ch}\,(B_1 L_1)}. \tag{14}$$

The symbols in (12) and (13) are

$$q_{sL}^*(p) = -\lambda_s \frac{\partial U^*(x,\ p)}{\partial x}\bigg|_{x=L_s+0};\quad q_{tL}^*(p) = -\lambda_t \frac{\partial U^*(x,\ p)}{\partial x}\bigg|_{x=L_s-0}.$$

As (6) is obeyed, the heat fluxes are equal at the boundary between the two bodies $x = L_s \pm 0$:

$$q_{sL}^*(p) = q_{tL}^*(p) = q_t^*(p). \tag{15}$$

We use the condition for equal temperature in the contact planes between the standard and test bodies:

$$U_s(L_s,\ t) = U_t(L_s,\ t)\ \text{and}\ U_s(0,\ t) = U_1(0,\ t),$$

which gives an equation for the parameter g_t, which is required to calculate a_t and λ_t:

$$\Phi(g_t,\ k) \equiv \frac{\mathrm{th}(\sqrt{g_t})}{\mathrm{th}(\sqrt{kg_t})} = \frac{1}{\Phi(g_s,\ k)}\left[\frac{\dfrac{q_s^*(p)}{q_t^*(p)\,\mathrm{ch}(\sqrt{g_s})} - 1}{\dfrac{q_s^*(kp)}{q_t^*(kp)\,\mathrm{ch}(\sqrt{kg_s})} - 1}\right] \equiv \Theta(p,\ k), \tag{16}$$

in which $g_t = pb^2/a_t$, $g_s = pL_s^2/a_s$ are dimensionless parameters, $\Phi(g,\ k) = \mathrm{th}(\sqrt{g})/\mathrm{th}(\sqrt{kg})$ is a special function, b the thickness in the test material, and L_s the thickness of the upper standard body.

From $q_s(t)$ and $q_t(t)$ one calculates the right-hand side in (16) $\Theta(p,\ k)$ and then one can use $\Phi(g_t,\ k) = \Theta(p,\ k)$ for preset k and p to determine g_t, e.g., by computer iteration. The result for g_t enables one to calculate the thermal diffusivity from

$$a_t = \frac{pb^2}{g_t}. \tag{17}$$

We derive λ from

$$\lambda_t = \lambda_s \frac{b}{L_s} \frac{\sqrt{g_s}}{\sqrt{g_t}} \mathrm{th}(\sqrt{g_s}) \frac{\mathrm{th}(\sqrt{g_t})}{\left[\dfrac{q_s^*(p)}{q_t^*(p)\,\mathrm{ch}(\sqrt{g_s})} - 1\right]}, \tag{18}$$

where g_s is known in advance, g_t is derived from (16), and the bulk specific heat in the test specimen is given by

$$C\gamma = \frac{\lambda_t}{a_t}. \tag{19}$$

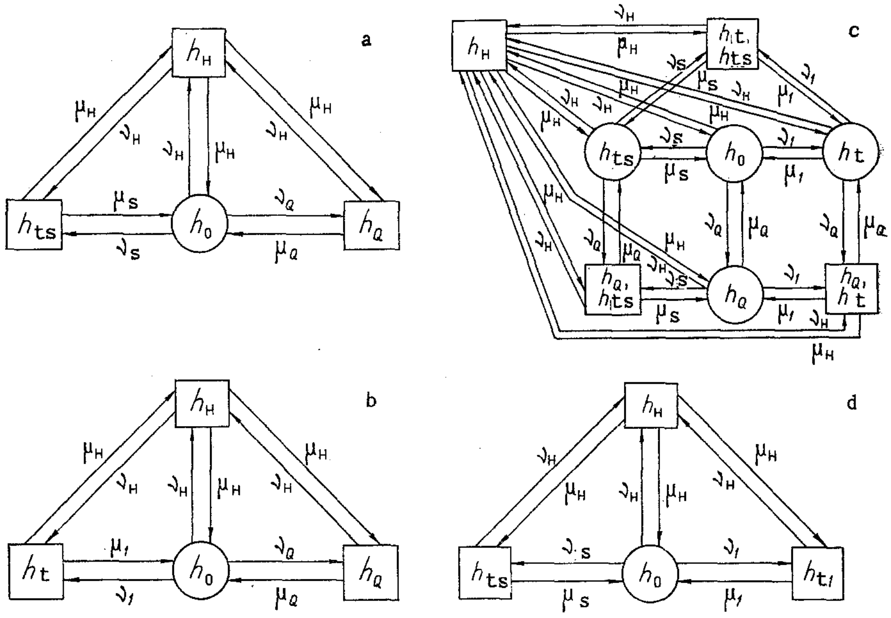

Fig. 2. Graphs for state change in measuring-instrument operation: a) w_1; b) w_2; c) w_3; d) w_4.

Then $q_s^*(p)$ and $q_t^*(p)$ are determined for (16) and (18) from the measured $U_s(\ell_s, t)$ and $U_1(\ell_1, t)$ together with Q(t).

We consider various methods of determining $q_s^*(p)$ and $q_t^*(p)$ in order to calculate the thermophysical characteristics from (17)-(19).

Method 1. Q(t) and $U_s(\ell_s, t)$, the temperature of the upper standard specimen in any plane $x = \ell_s$, are known, and then the formulas for the heat fluxes are

$$q_s^*(p) = \frac{Q^*(p)\dfrac{L_1}{\lambda_1\sqrt{g_1}}\operatorname{th}(\sqrt{g_1}) - \dfrac{U_s^*(l_s, p)}{\operatorname{ch}(\eta_s\sqrt{g_s})}}{\dfrac{L_1}{\lambda_1\sqrt{g_1}}\operatorname{th}(\sqrt{g_1}) + \dfrac{L_s}{\lambda_s\sqrt{g_s}}\operatorname{cth}(\sqrt{g_s}) - \dfrac{L_s}{\lambda_s\sqrt{g_s}}\dfrac{\operatorname{ch}(m_s\sqrt{g_s})}{\operatorname{ch}(\eta_s\sqrt{g_s})\operatorname{sh}(\sqrt{g_s})}}, \tag{20}$$

$$q_t^* = \frac{q_s^*(p)\operatorname{ch}(m_s\sqrt{g_s}) - U_s^*(l_s, p)\dfrac{\lambda_s\sqrt{g_s}}{L_s}\operatorname{sh}(\sqrt{g_s})}{\operatorname{ch}(\eta_s\sqrt{g_s})}. \tag{21}$$

In (20) and (21), $g_1 = PL_1^2/a_1$, $m_s = (L_s - \ell_s)/L_s$, $\eta_s = \ell_1/L_s$.

Method 2. Q(t) and the temperature in the lower standard specimen $U_1(\ell_1, t)$ are known, the latter for any plane in that body $x = -\ell_1$. Correspondingly,

$$q_s^*(p) = Q^*(p) - \frac{U_1^*(l_1, p)\operatorname{ch}(\sqrt{g_1})}{\operatorname{sh}(m_1\sqrt{g_1})}\frac{\lambda_1\sqrt{g_1}}{L_1}, \tag{22}$$

$$q_t^*(p) = Q^*(p)\operatorname{ch}(\sqrt{g_s}) - \frac{U_1^*(l_1, p)}{\operatorname{sh}(m_1\sqrt{g_1})}\left[\frac{\lambda_1\sqrt{g_1}}{L_1}\operatorname{ch}(\sqrt{g_1})\operatorname{ch}(\sqrt{g_s}) + \frac{\lambda\sqrt{g_s}}{L_1}\operatorname{sh}(\sqrt{g_1})\operatorname{sh}(\sqrt{g_s})\right]. \tag{23}$$

In (22) and (23), $m_1 = (L_1 - l_1)/L_1$, $\eta_1 = l_1/L_1$.

Method 3. One knows Q(t) together with $U_1(\ell_1, t)$ and $U_s(\ell_s, t)$, in which case one gets the simplest formulas for the fluxes:

$$q_s^*(p) = Q^*(p) - \frac{U_1^*(l_1, p)\operatorname{ch}(\sqrt{g_1})}{\operatorname{sh}(m_1\sqrt{g_1})}\frac{\lambda_1\sqrt{g_1}}{L_1}, \tag{24}$$

TABLE 3. Failure Rates ν in Measuring Instruments for Thermophysical Characteristics and Recovery Rates μ

Component	Frequency ν, h⁻¹			Recovery rate μ, h⁻¹	
	symbol	ν_{min}	ν_{max}	symbol	μ
Temp. PS at $x = \ell_s$	ν_s	$9\cdot 10^{-4}$	$3{,}6\cdot 10^{-3}$	μ_s	$4{,}2\cdot 10^{-2}$
Temp. PS at $x = \ell_1$	ν_1	$9\cdot 10^{-4}$	$3{,}6\cdot 10^{-3}$	μ_1	$4{,}2\cdot 10^{-2}$
Meter for Q(t)	ν_Q	$11{,}2\cdot 10^{-3}$	$4{,}4\cdot 10^{-2}$	μ_Q	$1{,}25\cdot 10^{-1}$
Heater	ν_H	$9\cdot 10^{-4}$	$3{,}6\cdot 10^{-3}$	μ_H	$4{,}2\cdot 10^{-2}$

$$q_t^*(p) = \frac{q_s^*(p)\,\mathrm{ch}(m_s\sqrt{g_s}) - U_s^*(l_s, p)\dfrac{\lambda_s\sqrt{g_s}}{L_s}\,\mathrm{sh}(\sqrt{g_s})}{\mathrm{ch}(\eta_s\sqrt{g_s})}. \quad (25)$$

Here $q_s^*(p)$ is analogous to (22) and $q_t^*(p)$ to (21).

<u>Method 4.</u> One knows $U_1(\ell_1, t)$ and $U_s(\ell_s, t)$, and the working relations are

$$q_s^*(p) = \frac{\dfrac{U_s^*(l_s, p)}{\mathrm{ch}(\eta_s\sqrt{g_s})} - \dfrac{U_1^*(l_1, p)\,\mathrm{sh}(\sqrt{g_1})}{\mathrm{sh}(m_1\sqrt{g_1})}}{\dfrac{L_s}{\lambda_s\sqrt{g_s}}\left[\dfrac{\mathrm{ch}(m_s\sqrt{g_s})}{\mathrm{ch}(\eta_s\sqrt{g_s})\,\mathrm{sh}(\sqrt{g_s})} - \mathrm{cth}(\sqrt{g_s})\right]}, \quad (26)$$

$$q_t^*(p) = \frac{q_s^*(p)\,\mathrm{ch}(m_s\sqrt{g_s}) - U_s^*(l_s, p)\dfrac{\lambda_s\sqrt{g_s}}{L_s}\,\mathrm{sh}(\sqrt{g_s})}{\mathrm{ch}(\eta_s\sqrt{g_s})}. \quad (27)$$

(20)-(27) contain the measurements of (10) and (11) on $U_1^*(\ell_1, p)$, $U_s^*(\ell_s. p)$ and the thermal power $Q^*(p)$; the numerical values are calculated from the measured $Q(t)$, $U_1(\ell_1, t)$ and $U_s(\ell_s, t)$, where the method of calculating the integral characteristics has been given [3, 7]. Analytic studies have shown that the optimum values for g are 0.08-0.14 for k = 12, with the optimality defined from the condition for least error in determining the thermal diffusivity and the best sensitivity.

These methods have been tested in an automated system based on an Iskra-1256 data-acquisition system, which can operate with various measuring instruments.

We examined the errors in the use of these methods of determining the flux characteristics; the errors were calculated by the [8, 9] method with the incorporation of the systematic, methodological, and instrumental sources for each.

Table 1 shows that the accuracy here decreases as the number of primary sensors (PS) is reduced and as one simplifies the instrument design, but the working formulas then become more complicated.

One thus has to choose a method and corresponding device design to suit the accuracy; implementation may involve other criteria such as reliability and economic performance, which together with the accuracy govern the over-all performance of the corresponding method and device.

We consider a technique for choosing methods and devices on the basis of a general criterion, which we take as one defining the accuracy and working features (failure rates for the PS and individual measurement units with recovery times). The analysis shows that with normal instrument operation, one has the following permissible forms:

$$W_p(h_0) = (w_1, w_2, w_3, w_4),$$

in which w_1, w_2, w_3, w_4 are the above methods of determining the heat flux.

The sets H_{w_i}, i = 1, ..., n of operating states are shown in Table 2, which indicates w_3 is viable not only in state h_0, where all the sensors are operational, but also in state

TABLE 4. Effective Measuring-Instrument Viability Parameters

Functional state	Performance parameter	Determination methods with various ways of measuring flux characteristics			
		w_1	w_2	w_3	w_4
h_0	$I_T(e, w, h)$	[0,852—0,892]	[0,883—0,913]	[0,902—0,929]	[0,871—0,907]
	$I(P, w, h, \frac{T}{h(0)})$	[0,7494—0,9268]	[0,7494—0,9268]	[0,7239—0,9191]	[0,9191—0,9788]
	$I(P, w, h)$	[0,6598—0,8862]	[0,6598—0,8862]	[0,5871—0,8621]	[0,7955—0,9396]
h_{ts}	$I_T(e, w, h)$	0	0	[0,871—0,907]	0
	$I(P, w, h, \frac{T}{h(0)})$	0	0	[0,0249—0,0077]	0
	$I(P, w, h)$	0	0	[0,0659—0,0244]	0
h_{t1}	$I_T(e, w, h)$	0	0	[0,852—0,892]	0
	$I(P, w, h, \frac{T}{h(0)})$	0	0	[0,0249—0,0077]	0
	$I(P, w, h)$	0	0	[0,0659—0,0244]	0
h_Q	$I_T(e, w, h)$	0	0	[0,883—0,913]	0
	$I(P, w, h, \frac{T}{h(0)})$	0	0	[0,1955—0,06]	0
	$I(P, w, h)$	0	0	[0,2091—0,0785]	0
	$E(w, H, \frac{T}{h(0)})$	[0,6385—0,8267]	[0,661—0,846]	[0,7192—1,0]	[0,799—0,887]
	$E(w, H)$	[0,5621—0,7905]	[0,582—0,809]	[0,64092—1,0]	[0,692—0,852]

h_{ts}, where the PS for the temperature at $x = \ell_s$ is faulty, and in state h_{t1}, where the PS for the temperature at $x = \ell_1$ has failed, as well as in state h_Q, when there is no information on Q. Methods w_1, w_2, and w_4 are viable only in state h_0. Figure 2 shows the functional state-change graphs. The set of vertices in a graph is set H_{w_i} (the vertices denoted by circles correspond to viable states and by squares to states of complete failure). The arcs characterize the transitions from one state to another over short intervals. The probabilities of these transitions are proportional to the failure rate ν and the recovery rate μ in the corresponding parts. The subscripts to the failure and recovery rates are the same as those used in Table 3, which gives the results from a study over three years on measuring-instrument element reliabilities.

Table 4 gives meter performance figures based on E, which for method w_i and set H_{w_i} in general is defined by an interval [10] because no exact values are available for the input data during design in many cases but one can state ranges. For example, state probabilities can be calculated from the failure rates given as upper and lower limiting values.

In our case, E for prolonged operation ($t \to \infty$) is

$$E(w, H) = \sum_{h \in H_w} I_T(e, w, h) I(P, w, h), \tag{28}$$

while for restricted use during [0, T] it is

$$E(w, H, T \mid h(0)) = \frac{1}{T} \sum_{h \in H_w}{}' I(e, w, h) \int_0^h I\left(P, w, h, t|h(0)\right) dt, \tag{29}$$

in which I(P, w, h) is the stationary probability range, I(P, w, h, t|h(0)) the state probability range at time $t \in [0, T]$ provided that the system is in state h(0) at t = 0, and $I_T(e, w, h)$ is the method accuracy (a range reciprocally related to the errors in determining a_t and λ_t in accordance with Table 1). For the form w^* optimal on set H,

$$E(w^*) = \max \{E(w),\ w \in W_p(H)\}. \tag{30}$$

In (30), E(w) is represented by indices defined by (28) and (29) together with the partial ordering $E(w_1) = [E_1^{\ell}, E_1^{u}] < E(w_2) = [E_2^{\ell}, E_2^{u}]$ if $E_1^{u} \leqslant E_2^{\ell}$ or $E(w_1) < E(w_2)$ if $E_1^{\ell} \leqslant E_2^{\ell}$ and $(E_1^{\ell} + E_1^{u}) < (E_2^{\ell} + E_2^{u})$; here the superscripts ℓ and u denote lower and upper. The probabilities have been calculated from the ν and the recovery rates (Table 3); the I(P, w, h, T|h(0)) contain information on the operation for 10 h.

The following conclusions are drawn from the E. When a thermophysical device is implemented and used in an automated system, the accuracy is not always the only criterion defining the design, the system composition, and the method of characteristic calculation. Table 4 shows that method No. 4 is preferable to No. 2, although the accuracy of No. 4 is less than that of No. 2, and is slightly inferior to No. 3, although that method is much simpler in design.

An automated system may be built and measuring instruments made on the basis of various working factors, which themselves are based on generalized performance parameters.

Our method of selecting measuring instruments on the basis of the state and functioning set can be used here.

NOTATION

x, spatial coordinate; t, time; U≡U(x, t), temperature; Q, heater thermal power, W/m^2; q, heat flux density; a and λ, thermal diffusivity and thermal conductivity; p, parameter in integral Laplace transformation; U_b, boundary temperature; L, specimen thickness; B, η, and m, special parameters characterizing specimen geometry; g, dimensionless working parameter; U^* and q^*, integral time characteristics for temperature and heat flux; ℓ, coordinate of temperature primary sensor; w, measuring instrument type; h, functional state; e, method accuracy; W, H, and E, sets; I, probability interval; ν and μ, failure and recovery rates; P, functional state probability.

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